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Some Applications of β * - Open Sets

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Abstract- In this paper, β*-open sets are used to define β*-US spaces and β* Urysohn spaces into pological spaces and study some of their basic properties.

Keywords- β*-open sets, β*-US space, β*-Urysohn space.

1. INTRODUCTION

The concept of US-spaces was first introduced by Slepian [3] and was further studied by Culler [2]. A topological space X is said to be a US- space if every sequence in X converges to a unique point. Generalized open sets play a very important role in General Topology and they are now the research topics of many topologists worldwide. Indeed a significant theme in General Topology and Real analysis concerns the various modified forms of continuity, separation axioms etc. by utilizing generalized open sets. Recently, Ali M. Mubarki et. al. [1] Introduced a new class of generalized open sets called β^*-open sets into the field of topology. In this paper, β^*-open sets are used to define β^*-*US* spaces and β^*-Urysohn spaces in topological spaces and study some of their basic properties.

2. PRELIMINARIES

For a subset A of a topological space (X, τ), Cl (A) and Int (A) denote the closure of A and the interior of A, respectively.

Definition 2.1. [4] The δ -closure of A, denoted by Cl_δ (A), is defined to be the set of all $x \in X$ such that $A \cap Int(Cl(U)) \neq \phi$ for every open neighbourhood U of X. If A = Cl_δ (A), then A is called δ -closed. The complement of a δ -closed set is a called δ -open set. The δ -interior of A is defined by the union of all δ -open sets contained in A and is denoted by Int_δ (A).

Definition2.2.[1]A sub set of a topological space (X,τ) is said to be β*-open if $S \subset Int(Cl(Int(S))) \cup Int(Cl_\delta(S))$. The complement of a β*-closed set is called a β*-open set. The family of all β*-open (β*- closed) subsets of (X,τ) is denoted by β*O(X)(β*C(X)). The family of all β*-open set of (X,τ) containing a point $x \in X$ is denoted by β*O(X, x).

Definition 2.3. [1] The intersection of all β*-closed sets containing $A \subset X$ is called the β*-closure of A and is denoted by β*Cl (A). The union of all β*-open sets contained in $A \subset X$ is called the β*-interior of A and is denoted by β*Int (A).

Definition 2.4. A topological space (X, τ) is said to be

(1) β*-T_1[1] iff or each pair of distinct point x *and* y of X, there exists β*-open sets and U and V such that $x \in U, y \notin U$ and $x \notin V, y \in V.$
(2) β*-T_2[1] iff or each pair of distinct point x *and* y of X, there exists β*- open sets U and V such that $x \in U, y \in V$ *and* $U \cap V = \phi$
(3) β*- R_1if for $x, y \in X$ with β*Cl({ x }) ≠ β*Cl ({ y }), there exist disjoint β*-open sets U and V such that β*Cl({ x }) ⊂ U and β*Cl({ y }) ⊂V.

Theorem 2.5. *Let (X, τ) be an topological space. Then (X, τ) β*is -T_2 if and only if it is β*-R_1 and β*-T_0.*

3. ON β^*-US-SPACES

Definition 3.1. A sequence (x_n) is said to be:
(1) β*- convergence to a point x *of* X , denoted by $(x_n) \xrightarrow{\beta^*} x$, *if* (x_n) is eventually in every β*-open set containing x.
(2) β*-US if every β*-convergent sequence (x_n) in X β*-converges to a unique point.

Clearly every US-space is β*-US but the converse is not true in general, as it can seen from the following example.

Example 3.2. Let X = {a,b,c} and τ= { ϕ,{a ,b} , X }.Then (X, τ) β*-US but not a US-space.

Theorem 3.3. *Every β*-T_2 space is β*-US.*

Proof. Let (X, τ) be a β*-T_2space and (x_n) be a sequence in X .Suppose that (x_n) β*-converges to two distinct points x and y. That is, (x_n) is eventually in every β*-open set containing x and also in every β*-open set containing y.This is a contradiction since(X,τ) is β*-T_2. This shows that the space (X, τ) is β*-*US*.

Theorem 3.4. *Every β*-US space is β*-T_1.*

Proof. Let (X,τ) be a β*-*US* space and x and y be two distinct points of X. Consider the sequence (x_n) where $x_n \neq x$ for every n. Clearly (x_n) β*-converges to x. Also, since $x \neq y$ and (X,τ) is β*-*US*,(x_n) cannot β*-converges to y, that is, there exists a β*-open set V containing y but not x. Similarly,if we consider the sequence(y_n) where $y_n \neq y$ for all n, and proceeding as above we get a β*-open set U containing x but not y. This shows that the space (X, τ) isβ*-T_1.

Definition 3.5. A subset S of (X, τ) is said to be:

(1) sequentially β*-closed if every sequence in S β*-converging in X β*-converges to a point in S.
(2) sequentially β*-compact if every sequence in S has a subsequence which β*-converges to a point in S.

Theorem3.6.*In a β*-US space, every sequentially β*-compact set is sequentially β*-closed.*

Proof. Let (X, τ) be a β*-*US* space and Y be a sequentially β*-compact subset of X.Let (x_n) be a sequence in Y . Suppose that (x_n) β*-converges to a point in X\Y. Let (x_{nk}) be a subsequence of (x_n) which β*-converges to a point $y \in Y$ since Y is sequentially β*-compact. Also, a subsequence (x_{nk}) of (x_n) β*-converges to $x \in$ X\Y. Since (x_{nk}) is a sequence in the β*-*US* space X, $x = y$. This shows that Y is sequentially β*-closed set.

Lemma3.7.[1]*Let A and X_0 be subsets of a topological space (X,τ). Then,*

(1) If $A \in \beta^* O(X)$ and X_0 is δ-open in (X, τ), then $A \cap X_0 \in \beta^* O(X_0)$;
(2) If $A \in \beta^* O(X)$ and X_0 is open in (X,τ), then $A \cap X_0 \in \beta^* O(X_0)$;
(3) If $A \in \beta^* O(X_0)$ and $X_0 \in \beta^* O(X)$, then $A \in \beta^* O(X)$.

Theorem 3.8. *Every δ-open subset of a -US space is β*-US.*

Proof. Let (X, τ) be a β*-*US* a topological space and Y be a δ-open subset of X. Let (x_n) be a sequence in Y. Suppose that (x_n) β*- converge to x and y in Y. We shall prove that (x_n) β*-converges to x and y in X. Let U be any β*-open subset of X containing x and V be any β*-open set of X containing y. *Then by Lemma 3.7,* $U \cap Y$ and $V \cap Y$ are β*-open sets in Y. Therefore, (x_n) is eventually in $U \cap Y$ and $V \cap Y$ and so in U and V. Since X is a β*-US space, this implies that $x = y$ and hence the subspace Y is a β*-US space.

Theorem3.9.*A topological space (X,τ) is β*-T_2 if and only if it is both β*-R_1 and β*-US.*

Proof. Let(X,τ) be a β*-T_2space.Then (X,τ) is β*-R_1 by Theorem2.5 and β*-*US* by Theorem 3.3. Conversely, let (X, τ) be both β*-$R_1$and β*-*US* space. By Theorem 3.4, we obtain that every β*-*US* space is β*-$T_1$and X is both β*-$T_1$and β*-R_1, it follows from Theorem 2.5 that (X, τ) is β*-T_2.

Lemma 3.10. [1] *The product of two β*-open sets is β*-open.*

Theorem3.11. *A topological space (X,τ) is β*-US if and only if the diagonal Δ is a sequentially β*-closed subset of X×X.*

Proof. Let (X,τ) be a β*-US space. Let (x_n, y_n) be a sequence in Δ. Suppose that (x_n, y_n) converges to (x, y).That is, (x_n) β-converges to x and y .Therefore, $x = y$ .Hence Δ is sequentially β-closed. Conversely, let Δ is sequentially β-closed. Let a sequence (x_n) β-converges to x and y

.Hence, (x_n, y_n) converges to (x, y) Since Δ is sequentially β-closed, $(x, y) \in \Delta$, which means that $x = y$.

Definition3.12. A function f:(X,τ)→(Y,σ) is said to be strongly β*-open (resp. strongly β*-closed) if f (A) $\in\beta$*O(Y) (resp. f (A)$\in\beta$*C(Y)) for every A $\in\beta$*O(X) (resp. A $\in\beta$*C(X)).

Lemma3.13. *Let a bijection f:(X,τ)→(Y,σ) is strongly β*-open. Then for any $A \in \beta^*C(X)$, $f(A) \in \beta^*C(Y)$*

Theorem3.14.*The image of a β*-US space under a bijective strongly β*-closed is β*-US.*

Proof. Let $f : (X,\tau) \to (Y,\sigma)$ is strongly β*-closed function and let (X, τ) be a β*-*US* space. Let (y_n) be a sequence in Y .Suppose that (y_n) β-converges to x and y. In that case, we shall prove that the sequence $(f^{-1}(y_n))$ β-converges to $f^{-1}(x)$ and $f^{-1}(y)$.Let $U \in \beta^* O(X, f^{-1}(x))$.*Then* $f(U) \in \beta^* O(X, x)$ and hence (y_n) is eventually in $f(U)$.Therefore,$(f^{-1}(y_n))$ eventually in U .Hence,$(f^{-1}(y_n))$ β-converges to $f^{-1}(x)$. Similarly, we can prove that $(f^{-1}(y_n))$ β-converges to $f^{-1}(y)$.This is not possible, since (X,τ) is β*-*US* space. Hence,(Y,σ) is β*-*US*.

Definition 3.15. A function f : (X, τ) → (Y,σ) is said to be:

(1) sequentially β*-continuous at $x \in X$ if f (x_n) β*-converges to f(x) whenever (x_n) is a sequence β*-converging to x ;

(2) Sequentially β*-continuous iff is sequentially β*-continuous at all $x \in X$;

(3) Sequentially nearly β*-continuous if for each point $x \in X$ and each sequence (x_n) in X β*-converging to x ,there exists a sub-sequence(x_{nk}) of (x_n) such that $f(x_{nk}) \underset{\to}{\beta^*} f(x)$;

(4) Sequentially sub β*-continuous if for each point $x \in X$ and each sequence (x_n) in X β*-converging to x ,there exists a subsequence of(x_{nk}) of (x_n) and a point $y \in$ Y such that $f(x_{nk}) \underset{\to}{\beta^*} y$;

(5) Sequentially β*-compact preserving if the image f(K) of every sequentially β*-compact set K of X is sequentially β*-compact in Y.

Theorem 3.16. *Let f: $(X,\tau) \to (Y, \sigma)$ and g:Y $\to Z$ be two sequentially β*-continuous functions .If Y is β*-US, then the set $A = \{x : f(x) = g(x)\}$ is sequentially β*-closed.*

Proof. Let Y be a β*-*US* space and suppose that there exists a sequence (x_n) in A β*-converging to $x \in X$ .Since f and g are sequentially β*-continuous functions, $f(x_n) \underset{\to}{\beta*} f(x)$ and $g(x_n) \underset{\to}{\beta*} g(x)$. Hence $f(x) = g(x)$ and $x \in A$. Therefore, we obtain A is sequentially β*-closed.

Theorem 3.17. *Every function f: $(X,\tau) \to (Y, \sigma)$ is sequentially sub- β*-continuous if Y is sequentially β*-compact.*

Proof. Let (x_n) be a sequence in X β*-converging to a point x of X. Then $(f(x_n))$ is a sequence in Y and as Y is sequentially β*-compact, there exists a subsequence $(f(x_{nk}))$ of $(f(x_n))$ β*- converging to a point $y \in Y$. This shows that f: $X \to Y$ is sequentially sub- β*-continuous.

Theorem3.18. *Every sequentially nearly β*-continuous function is sequentially β*-compct preserving.*

Proof. Suppose that f: (X, τ) →Y is sequentially nearly β*-continuous function and let A be any sequentially β*-compact set of Y. Let (y_n) be any sequence in $f(A)$. Then for each positive integer n, there exists a point $x_n \in A$ such that $f(x_n) = y_n$. Since (x_n) is a sequence in the sequentially β*-compact set A, there exists a subsequence (x_{nk}) of (x_n) β*-converging to a point $x \in A$. Since f is sequentially nearly β*-continuous, then there exists a subsequence (x_j) of (x_{nk}) such that $f(x_j) \underset{\to}{\beta*} f(x)$. Therefore, there exists a subsequence (y_j) of (y_n) β*- converging to $f(x) \in f(A)$. This shows that $f(M)$ is sequentially β*-compact set in Y.

Theorem 3.19. *Every sequentially β*-compact preserving functionis sequentially sub-β*-continuous.*

Proof. Suppose $f : X \to Y$ is a sequentially β*-compact preserving function. Let x be any point of X and (x_n) any sequence in X β*-converging to x. We shall denoted the set $\{x_n: n = 1,2,. \}$ by A and $B = A \cup \{x\}$. Then B is sequentially β*-compact Since $x_n \overset{\beta^*}{\to} x$. Since f is sequentially β*-compact set preserving, it follows that $f(B)$ is a sequentially β*-compact set of Y. Since $(f(x_n))$ is a sequence in $f(B)$, there exists a subsequence $(f(x_{nk}))$ of $(f(x_n))$ β*-converging to a point $y \in f(B)$. This implies that f is sequentially sub- β*-continuous.

Theorem 3.20. *A function $f : (X, \tau) \to (Y, \sigma)$ is sequentially β*- compact preserving if and only if $f_{|M}: M \to f(M)$is sequentially sub- β*-continuous for each sequentially β*-compact subset M of X.*

Proof. Suppose that $f:(X,\tau)\to(Y,\sigma)$ is a sequentially β*-compact preserving function. Then $f(M)$ is sequentially β*-compact set M of X. Therefore, by Theorem 3.17, $f_{|M}: M \to f(M)$ is sequentially sub- β*- continuous function. Conversely, let M be any sequentially β*-compact set in Y .We shall show that $f(M)$ is sequentially β*-compact set in Y . Let (y_n) be any sequence in $f(M)$. Then for each positive integer n, there exists a point $x_n \in M$ such that $f(x_n) = y_n$. Since (x_n) is a sequence in a sequentially β*-compact set M , there exists a subsequence (x_{nk}) of (x_n) β*-converging to a point $x \in M$ .Since $f_{|M}: M \to f(M)$ is sequentially sub- β*-continuous, there exists a subsequence (y_{nk}) of (y_n) β*-converging to a point $y \in f(M)$. This implies that $f(M)$ is sequentially β*-compact set in Y . Thus,$f: X \to Y$ is sequentially β*-compact preserving function.

The following theorem gives a sufficient condition for a sequentially sub- β*-continuous function to be a sequentially β*-compact preserving.

Theorem3.21. *If a function $f: (X, \tau) \to (Y, \sigma)$ is sequentially sub- β*-continuous and $f(M)$ is sequentially β*-closed set in Y for each sequentially β*-compact set M of X, then f is sequentially β*-compact preserving function.*

Proof. We use the previous Theorem. It suffices to prove that $f(\backslash M : M \to f(M))$ is sequentially sub-β*-continuous for each sequentially β*- compact subset M of X. Let (x_n) be any sequence in M β*-converging to a point $x \in M$. Then since f is sequentially sub-β*-continuous, there exists a subsequence (x_{nk}) of (x_n) and a point $y \in Y$ such that $f(x_{nk})$ β*-converges to y. Since $f(x_{nk})$ is a sequence in the sequentially β*-closed set $f(M)$ of Y,we obtain $y\in f(M)$.This implies that $f_{|M}:M\to f(M)$ is sequentially sub-β*-continuous.

4. β*-URYSOHNSPACES

Definition 4.1. An topological space (X,τ) is said to be β*-Urysohn if for each pair of distinct points x and y in X, there exist $U \in \beta^* O(X,x)\, and\, V \in \beta^* O(X,y)$ such that β*Cl(U)∩β*Cl(V)= ϕ.

Remark4.2.*Every Urysohn space is β*-Urysohn.But the converse is not true in general, as shown by the following example.*

Example 4.3.*Let $X= \{a,b,c\}$ and $\tau = \{\phi$,{a},{b,c},X}.Then (X, τ) is β*- Urysohn but not Urysohn.*

Theorem 4.4. *A β*-Urysohn space is β*-T_1.*

Proof. Let x and y be any two distinct points of X. Since (X, τ) β*- Urysohn, there exist $U \in \beta^* O(X,x), V \in \beta^* O(X,y)$ such that $\beta^* Cl(U) \cap \beta^* Cl(V) = \phi$ .Then $x \notin \beta^* Cl(V)\, and\, y \notin \beta^* Cl(U)$. Now,$\beta$*Cl($U$),$\beta$*Cl($V$)$\in\beta$*$C(X)$. Therefore, $X\backslash \beta$*Cl(U), $X\backslash \beta$*Cl(V)$\in\beta$*$O(X)$ and are such that $x \in X\backslash \beta$*Cl(V) and $y \in X\backslash \beta$*Cl(U) while $x \notin X\backslash \beta$*Cl($U$) and $y \notin X \backslash \beta^* Cl(V)$ .Thus,(X,τ) is β*-T_1.

Theorem4.5. *Every δ-open subset of a β*-Urysohn space is β*- Urysohn.*

Proof. Let Y be a δ-open subset of X and x, $y \in Y\subset X$ such that $x \neq y$. Since (X, τ) is β*-Urysohn, there exist $U \in \beta^* O(X,x), V \in \beta^* O(X,y)$ suchthat $\beta^* Cl(U) \cap \beta^* Cl(V) = \phi$ .Since Y is δ-open,$U\cap Y\in \beta*_Y O(X,x)$,$V\cap Y\in \beta*_Y O(X,y)$.Also $\beta*_Y$ Cl($U\cap Y$)∩ $\beta*_Y$ Cl($V\cap Y$)=(β*Cl($U\cap Y$)∩Y)∩ (β*Cl($V\cap Y$)∩Y)=β*Cl($U\cap Y$)∩β*Cl($V\cap Y$)∩$Y\subset$ β*Cl(U)∩β*Cl(V)= ϕ .Therefore, $\beta*_Y$ Cl($U\cap Y$)∩ $\beta*_Y$ Cl($V\cap Y$)= ϕ .This indicates that (Y,σ) is an β*-Urysohn space.

β*-Urysohn space remain invariant under certain bijective function as is shown in the following theorem.

Theorem 4.6. *If a bijection $f : (X, \tau) \to (Y, \sigma)$ is strongly β*-open and (X, τ) is β*-Urysohn, then (Y, σ) is β*-Urysohn.*

Proof. Let y_1 and y_2 be any two distinct points of Y. Since f is bijective, $f^{-1}(y_1)$ and $f^{-1}(y_2)$

are two distinct points of X. Since (X, τ) is β*-Urysohn, there exist $U \in \beta^*O(X, f^{-1}(y_1))$, $V \in \beta^*O(X, f^{-1}(y_2))$ such that $\beta^*Cl(U) \cap \beta^*Cl(V) = \phi$. By the hypothesis of f, we have $f(\beta^*Cl(U)) \in \beta^*C(Y)$, since $\beta^*Cl(U) \in \beta^*C(X)$. It follows that $\beta^*Cl(f(U))$ $\subset f(\beta^*Cl(U))$. In a similar manner, $\beta^*Cl(f(V))$ $\subset f(\beta^*Cl(V))$. Then, by the injectivity of f, $\beta^*Cl(f(U)) \cap \beta^*Cl(f(V)) = \phi$ and hence (Y, σ) is β*-Urysohn.

Definition 4.7. A function $f : (X, \tau) \to (Y, \sigma)$ is said to be quasi β*-irresolute if for each $x \in X$ and for each $V \in \beta^*O(Y, f(x))$ there exists $U \in \beta^*O(X, x)$ such that $f(U) \in \beta^*Cl(V)$.

Theorem 4.8. *If (Y,σ)-Urysohn and $f : (X,\tau) \to (Y,\sigma)$ is strongly β*-irresolute, then (X, τ) is β*-T_2.*

Proof. Since f is injective, for any pair of distinct points $x_1, x_2 \in X$, $f(x_1) = f(x_2)$. The β*-Urysohn property of Y indicates that there exist $V_i \in \beta^*O(X, f(x_i))$, $i=1,2$ such that

$\beta^*Cl(V_1) \cap \beta^*Cl(V_2) = \phi$. Hence

$f^{-1}(\beta^*Cl(V_1)) \cap f^{-1}(\beta^*Cl(V_2)) = \phi$. Since f is strongly β*-irresolute, there exists

$U_i \in \beta^*O(X, x_i)$, $i=1, 2$ such that $f(U_i) \subset \beta^*Cl(V_i)$, $i=1, 2$. It then follows that $U_i \subset f^{-1}(\beta^*Cl(V_i))$, $i=1,2$. Hence $U_1 \cap U_2 \subset f^{-1}(\beta^*Cl(V_1)) \cap f^{-1}(\beta^*Cl(V_2)) = \phi$. This implies that (X, τ) is β*- T_2.

Definition 4.9. An topological space (X,τ) is said to be β*-regular[1] if for each $F \in \beta^*C(X)$ and each $x \notin F$, there exist disjoint β*-open sets U and V such that $x \in U$ and $F \subset V$.

Theorem 4.10. [1] *For a topological space (X,τ), the following properties are equivalent:*

1. *(X, τ) is β*-regular;*
2. *For each $U \in \beta^*O(X)$ and each $x \in U$, there exists $V \in \beta^*O(X)$ such that $x \in V \subset \beta^*Cl(V) \subset U$.*

Theorem 4.11. *A β*-regular β*-T_2 space is β*-Urysohn.*

proof. Let (X, τ) be β*-regular β*-T_2 space. Since (X, τ) is β*-T_2 for any pair of distinct points $x_1, x_2 \in X$, there exist $U \in \beta^*O(X, x_1)$ and $V \in \beta^*O(X, x_2)$ such that $U \cap V = \phi$. Now, $X \backslash \beta^*Cl(U)$ is β*-open in (X, τ) containing x_2. The β*-regularity of X gives the existence of a $W \in \beta^*O(X, x_2)$, by Theorem 4.10, such that $x_2 \in W \subset \beta^*Cl(W) \subset X \backslash Cl(U)$. This implies that $\beta^*Cl(U) \cap \beta^*Cl(W) = \phi$. It follows that (X,τ) is β*-Urysohn.

Theorem 4.12. *The product of two β*-Urysohn spaces is β*-Urysohn.*

Proof. Follows from Lemma 3.10.

Definition 4.13. *A graph $G(f)$ of a function $f : (X, \tau) \to (Y, \sigma)$* is said to be strongly β^*-closed if for each $(x, y) \in (X \times Y) \backslash G(f)$, there exist $U \in \beta^*O(X, x)$ and $V \in \beta^*O(Y, y)$ such that $(U \times \beta^*Cl(V) \cap G(f) = \phi$.

Lemma 4.14. *A graph $G(f)$ of a function $f : (X,\tau) \to (Y, \sigma)$ is strongly β*-closed in $X \times Y$ if and only if for each $(x,y) \in (X \times Y) \backslash G(f)$, there exist $U \in \beta^*O(X, x)$ and $V \in \beta^*O(Y, y)$ such that $f(U) \cap \beta^*Cl(V) = \phi$..*

Proof . It is an immediate consequence of Definition4.13.

Theorem 4.15. *If $f : (X, \tau) \to (Y, \sigma)$ is quasi β*-irresolute and $(Y, \sigma, \Im)$ is β*-Urysohn, then $G(f)$ is quasi β*-closed in $X \times Y$.*

Proof . Let $(x, y) \in (X \times Y) \backslash G(f)$, then $f(x) \neq y$. Since $(Y, \sigma, \Im)$ is β*-Urysohn, there exist $V \in \beta^*O(Y, y)$, $W \in \beta^*O(Y, f(x))$ such that $\beta^*Cl(V) \cap \beta^*Cl(W) = \phi$. Since f is quasi β*-irresolute, there exists $U \in \beta^*O(X, x)$ such that $f(U) \in \beta^*Cl(W)$. This, therefore, implies that $f(U) \cap \beta^*Cl(W) = \phi$. So, by Theorem 4.15 $G(f)$ is quasi β*-closed in $X \times Y$.

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